HEGEL AND THE MATHEMATICS COMMUNITY: A LEFT-SIDE HISTORY

HEGEL E A COMUNIDADE MATEMÁTICA: UMA HISTÓRIA PELA ESQUERDA

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Abstract: Starting from a proof of the fundamental theorem of calculus accessible to K-12 students, we apply Hegel's *Science of Logic* to Barrow's theorem. This article may also be considered as an introduction to speculative philosophy, adequate for mathematics educators. We focus on the subsection *Barrier and Ought,* where Hegel twists Kant's aphorism *you can because you ought* and obtains a precept of action aimed at infirming conservative political positions. We direct Hegel's *Ought* to criticize the pedagogical conservatism of the *twentieth century mathematics* (M20) community and its consequences to mathematics education. From the development of the article we elicit the concept of *speculative mathematics* as a political agenda for mathematics education.

Keywords: Fundamental theorem of calculus; Barrow's theorem; Hegel's Logic; Speculative philosophy; Mathematics community.

Resumo: A parir de uma demonstração do teorema fundamental do cálculo, acessível ao ensino médio, aplicamos a *Ciência da Lógica* de Hegel ao teorema de Barrow. O artigo também pode ser considerado como introdução à filosofia especulativa, adequada a educadores matemáticos. Focalizamos a subseção *Barrier and Ought* (*Barreira e Dever*), onde Hegel altera o aforismo kantiano *podes porque deves* e obtém um preceito para ação dirigido a abalar posições políticas conservadoras. Valemo-nos do *Dever* em Hegel para criticar o conservadorismo da comunidade de *matemática do século vinte* (M20) e suas consequências para a educação matemática. A partir do desenvolvimento do artigo, inferimos o conceito de *matemática especulativa* como agenda política para a educação matemática.

Palavras-chave: Teorema fundamental do cálculo; Teorema de Barrow; Lógica de Hegel; Filosofia especulativa; Comunidade matemática.

1 Introduction

Section 2 displays a proof of the fundamental theorem of calculus, accessible to K-12 students. This article results from the live discussion raised by the presentation of that proof in two discussion lists[2]. It was originally designed as a series of comments to the proof, cast into a set of pages that could be read in any order, each limited to five to six hundred words. This design could not be maintained, however, and section 7 ended up forming the backbone of this entire article.

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In section 3 we answer the question of why calculus emerged during the 17th century in Europe. Labor-power, a new commodity emerging in the wake of capitalism, required both standardization and cosmetic make-up in order to be presented to the market as perfect, just like any other commodity. Besides, what Marx (1818-1883) later called exploitation was already latent and needed justification. Ethical certainty became a social necessity; mathematicians became prominent as guardians of certainty. A lengthy discussion o speculative philosophy in connection to sociology and economy can be found in Coombs (2015).

In section 4, we introduce the reader to speculative philosophy as a permanent struggle to transcend the philosophies prevalent in the time of Hegel (1770-1883), especially Kant's critical philosophy. This struggle is part of the millenary debate between the so-called philosophers and those other philosophers called sophists. Hegel refers to the struggle of reason to transcend understanding. The debate occurs in the agora.[3] Hegel does not use this term, but we have introduced it and it has functioned as a concept in Hegel's Logic, making it easier to understand.

In section 5 we conjecture about how Hegel developed the motivation to design and practice speculative philosophy. Before getting his fist university position, he had spent eight years as a tutor in rich family homes, a position he only left reluctantly. Understanding turns with *hate and fury against reason*, he complains. This section explains some of the dichotomies to which understanding remains attached, especially the paradigmatic one, A=A, and elicits the conservative political role of understanding.

In section 6 we take a fragment from Wikipedia as an example of discourse of understanding, and proceed to exercise speculative philosophy on its reading. The main operation of speculative philosophy is to show how the enunciation of a proposition denies the content of its statement. This effect of language cannot be avoided. Examples of understanding dichotomies follow. Utterances trigger a movement of mediation in the agora that Hegel calls *becoming*. This is negation of negation.

Section 7 is structured upon a line by line reading of two paragraphs from a subsection of Hegel's Logic. In this section, we replace "finite something" with

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[3] In Ancient Greece the agora was initially the communities' meeting place to decide about laws. Later on, it became a marketplace and a stage for debates between philosophers and sophists. We will use "agora" to mean the channels through which one can express opinions and debate ideas today, including print and online publications, as well as discussion lists, online meetings, etc. Accordingly, we maintain that at this exact moment the reader is alongside us *in the agora*.

"Barrow's theorem," with three purposes: first, to show that once we find a concrete referent to Hegel's discourse, it becomes perfectly clear; second, to follow the logical development from Barrow's theorem up to the simplified version of the fundamental theorem of calculus of section 2; and third, to ground speculative mathematics on the precept of action that Hegel derives from Logic and that, twisting the meaning that it has in Kant, he calls *Ought.*

Is section 8 we present a proof of Barrow's theorem, remaining true to his thought and to his form, but transposing his text into modern notation and justifying all intermediate steps. The section also elicits the aspects of the proof that were mentioned in the previous section.

Section 9 is crucial to understand the concept of quantum of which Hegel says that, "in its complete determinateness is number" (HEGEL, 1966, p. 217). Two meanings of the word infinite (*Unendlich*) are here distinguished. Quantum, for instance, the length of the diagonal of a square with sides of length one, is *intrinsically infinite* in its concept, determined by the infinity of its otherness, that is, the magnitudes that lie across its limit. However, infinite quantities are not quanta. Quanta are infinite as a concept and finite as quantity.

Section 10 elucidates the difference between the bad and the true infinites. It is cast into the form of a dialogue between two hypothetical characters: "Hegel" and "John Q. Understanding". These characters also appear in other places of the paper, for instance, in the section 9, when both are busy weighing a slice of mortadella. These dialogues are designed to underscore the fact that speculative philosophy is a practice in the agora that includes the moment when the reader goes through these lines. Speculative philosophy has no externality. This is why it is *the* system, based on the *science of language*, namely, on Hegel's logic.

Section 11 describes the constitution of M20, starting from the turning point where Cauchy (1789-1857) and later Weierstrass (1815-1897) broke apart from the polemic about the foundations of calculus and founded a closed sub-commission in the agora. They locked out both true infinite and number, and, out of the bad infinite, developed the cornerstone of M20 community's criterion to decide what counts as a valid statement. The M20 community does not know what a number is, much less what it is that they call "mathematics".

In section 12 we introduce speculative mathematics as a criticism of the M20 community from the point of view of the true infinite.

In section 13 we collect the main result from the paper: speculative mathematics as a political agenda for the mathematics classroom.

2 The mortadella theorem

On May 30, 2020 we introduced the following proof of the fundamental theorem of calculus into two academic discussion lists. The text ended with challenging questions that generated an interesting discussion. That discussion provided us with the drive to write this article.

Take a piece of mortadella and ask the grocer to cut it into equally thin slices without breaking their order. Put the sliced mortadella stacked on a table, taking care not to change the order of the slices. Then weigh each slice, proceeding sequentially from one end of the stack to the other. Register its weight in one column while, on a second column, add each of the weights to get the *accumulated weight* up to each line.

If you take the accumulated weight on any given line and subtract the accumulated weight on the previous line, you get the weight of the slice registered on that line. Now, let's get fancy: we express this by saying that the *differential of the accumulated weight is the weight of the slice*. Or, if you prefer, a little fancier: the weight of the slice is its area A, times its thickness dx, times the density of the mortadella, which is nearly one, so the weight[4] of the slice is $dW = A\,dx$. Dividing by dx we get $\dfrac{dW}{dx} = A$, and we say *that the derivative of the accumulated weight is the area of the slice*.

Why did it take humanity over than two thousand years do add "fancy" to a result accessible to a K-12 youth? Why did this only happen in the 17 century, given that areas and ratios had been worked out since Ancient Greece? What is it that was so achieved? This result is known as the fundamental theorem of calculus. Adding the weights $dW = A\,dx$ of the slices, from the first up to a slice distant x from the first one, we get the accumulated weight up to x. This sum is written with the symbol $\int$, a stretched version of our S, and called the integral of the area.

$$W(x) = \int_{x=0}^{x} A(x)\,dx.$$

With this notation the fundamental theorem of calculus becomes:

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[4] People in Brazil are used to expressing weight in kg-force, so that it becomes numerically equal to density times volume.

$$\frac{dW}{dx} = \frac{d}{dx} \int\limits_{x=0}^{x} A(x)\,dx = A(x)$$

In summary, this theorem says that *the derivative of the integral is the function itself*. Indeed, this seems quite a simple result. It seems simple because we have hidden its difficulty behind human *perception*, a category worked out by Edmund Husserl (1859-1938). From the experience people have with bologna sandwiches, no one doubts that a slice on a table is a disc whose volume may be calculated by the formula for the volume of the cylinder. The fact that the surrounding skin of the slice does not form a right angle with the table, especially at the ends of the piece, is preliminarily neglected *at the level of perception*. The formula for the volume of a right cylinder does not truly apply. With this trick, we skipped over two hundred years of logical development. We have induced the reader either to neglect infinitesimals of higher order or to assume the so-called "passing to the limit" that comprises the battle horse of M20.

There is a huge amount of bibliography about the current traditional history of calculus that virtually ignores what Hegel said about the mathematicians' attempts to justify their method. In this article we will endeavor to show that Hegel's speculative philosophy can lead to an entirely new history of the M20 community, with significant consequences to mathematics education.

3 Why in the 17th century?

Why did calculus originate in the 17th century, and neither sooner nor later, since it only required thought and writing? In trying to answer this question, we should avoid the ideology of "suddenly". "Suddenly," problems of *motion* started being studied by two geniuses: Galileo (1564-1642) and soon after, Newton (1643-1727); "suddenly" another genius, Leibniz (1646-1716), invented a *handy notation* for calculus; "suddenly" an ideologist of education, Comenius (1572-1670), started travelling across Europe, proposing *universal education*; "suddenly" a philosopher, Adam Smith (1723-1790), started talking about the *invisible hand* that turns individual self-interest into benefit for all; "suddenly" many scientists became interested in a new method of calculus, etc. We take the stand that these people did not become famous for what they said, but rather because they *found listeners to their speech* in a two-way feedback process: the more they talked, the more they were listened to, and vice versa. Now our question becomes clearer

and easier to answer: why did this *selective listening demand* start in Europe in the 17ᵗʰ century?

Here we rely on Sohn-Rethel's (1978) concept of *real abstraction*: what people do in their everyday activities in the sphere of production, predisposes them to listen to discourses that universalize their practices, simultaneously ensuring their ethical support. Already in Antiquity, the millenary practice of exchanging and selling led traders to present their commodities in a way that made them look perfect to the eyes of the buyer[5]. From time to time, this practice was ideologically confirmed by offering equally perfect and spotless victims to be sacrificed at the temples.

Material conditions changed with the blocking of the terrestrial route to the Middle East and the subsequent "discovery" of the New World. In the prosperous cities of Italy, *capital started moving and circulating by itself*, motivated only by interest. According to Sohn-Rethel (1978), this abstract movement through nothing sharpened people's ears to the universalized movement through abstract space considered by Newton. A single principle justified Kepler's empirical work. It is well known that Newton was always concerned about the opportunity to find attentive ears to his publications.

In the beginning of the 17ᵗʰ century, a commodity that had existed for millennia started gaining importance in the market in Europe: *labor power*. It had a price, the salary, and its use provided more wealth than its cost. In a society that condemned interest, this operation must have raised some moral inquietude, a reaction that was repressed for two hundred years, until Marx called it exploitation.

> Society is always traversed by an antagonistic split which cannot be integrated into the symbolic order. And the stake of social ideological fantasy is to construct the vision of a society that does exist, a society that is not split by an antagonistic division, a society in which the relation between its parts is organic and complementary" (Žižek, 1999, p. 126).

With the introduction of labor-power, the social split took on quantitative numerical form, creating the condition for the discourse of mathematicians to be heard in hopes of ensuring certainty about the new social economic procedure.

Also, labor-power, as any other commodity, should have its standardized pattern, its cosmetic make-up so as to look perfect, before being put up for sale. Therefore, Comenius was bound to find listeners. It is not surprising that "in his *Didactica Magna* (COMENIUS, 2001) he outlined a system of schools that is the exact counterpart

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[5] This practice persists today, when we choose our nutrition according to its appearance and disregard how much glyphosate it contains.

of the current American system of kindergarten, elementary school, secondary school, college, and university".[6] Is it just a "counterpart," though? How could he have guessed?

4 The endless struggle for speculative philosophy

Hegel praised mathematicians up to his time for their achievements, but criticized them for not having been able to justify their method through the notion of *true infinity* which underlies mathematical infinity, since "it is far superior to the ordinary so-called *metaphysical infinite* on which are based the objections to the mathematical infinite" (HEGEL, 1969, p. 241).

The mathematical infinite 'is commonly determined as a magnitude greater than which there is no greater" (HEGEL, 1966, p. 258)[7] whereas the metaphysical and the spurious infinite is the monotone race of jumping from 1st to 2nd, from 2nd to 3rd and so on, trying to reach an infinity that escapes and reconstitutes itself at each step. Hegel calls this the *infinite progress* of the *bad infinite*.

The true infinity, according to Hegel, is the *demonstration* of how understanding, as he called the philosophies of his time, constitutes itself by holding the bad infinity of space and time as absolute truth. Such demonstration occurs during the public philosophical debate in what we call *the agora*.[8] From the market square in Ancient Greece, the agora has now developed into hard-print, and on-line publications, as well as social interactive media, including academic journals. Together with the reader, we are now in the agora. Here, where we are, is the *absolute*, the place where speculative philosophy practice occurs. *The absolute is the agora*.

The true infinite is the repetition of a constant pressure, insofar as speculative philosophy acts to push understanding beyond itself and it reacts back, sticking to unilateral views; insofar as speculative philosophy tries to elicit the processes of which understanding only sees one of two poles. However, this second repetition occurs in the agora; it is an operation in the realm of discourse. Language must here be understood as including all manifestations of human subjects, including gestures, looks, smiles and silences. We consider that language is what Hegel calls mediation.

The practice of speculative philosophy is not without friction. Understanding, says Hegel, "must come to hate speculation when it has experienced it; and unless it is in the

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[6] https://en.wikipedia.org/wiki/John_Amos_Comenius
[7] We will use either of two English translations of Hegel's Logic, according to the adequacy of some words to our purpose.
[8] This is not a concept in Hegel.

state of perfect indifference that security confers, it is bound to detest and persecute it" (HEGEL, 1801, sec. 2, p. 8). It is peculiar of speculative philosophy that it only exists while in this struggle to push understanding beyond itself, trying to transcend (*aufheben*) it. Contrarily to understanding, speculative philosophy does not exist by itself. Adhesion of understanding to it would be the end of both. Indeed, in 1801, when Hegel referred to understanding as "common sense", he wrote:

> If common sense could grasp this scope [of speculation], it would not believe speculation to be its enemy. For in this higher synthesis of conscious and non-conscious, speculation also demands the nullification of conscious itself. Reason thus drowns itself (…) in its own abyss: and in this night (…) which is the noonday of life, common sense and speculation can meet each other. (HEGEL, 1801, sec. 2, p. 10).

Therefore, we should place Hegel's plea for a conceptual justification of mathematical procedures in the context of a perpetual asymmetrical struggle of reason, not against, but traying to transcend (*aufheben*) understanding. However, the effectiveness of our practice of speculative philosophy may be hampered by "the state of perfect indifference that security confers" to the M20 community in the closed form of its present certainty. Consequently, as a preliminary operation, we must bring the M20 community into the agora, so that we can examine the fabric of its certainty.

Thus far, we have tried to make clear what we should understand by speculative philosophy that in that 1801 book Hegel called reason. However, to make sense of "speculative mathematics" we must first decide what we mean by "mathematics".

5 What moved Hegel

What did move Hegel to design, realize and practice speculative philosophy? The project was ready in 1801, when Hegel obtained a non-salaried position (*Privatdozent*) at University Jena and started living as a boardinghouse mouse; he was 31 years old. He had spent eight years tutoring adolescents in rich family homes at Bern and Frankfurt, a position that he quit after misunderstandings with his patrons. In an 1801 book he asks what happens if Reason is powerless against the dichotomies of understanding, which at that time he called intellect. How does the intellect react? "Dichotomy felt itself attacked, and so turned with hate and fury against Reason, until the realm of the intellect rose such power that it could regard itself as secure from Reason" (HEGEL, 1801, sec. 2, p. 5).

He is harsh against the philosophies of his time, mainly those derived from Kant, This philosophy declares the impotence of reason to know the truth, namely the "thing-in-itself" and cleaves concepts into black/white, like finite and infinity, identity and

difference, either A=A or A≠A. Hegel argues that by saying A=A, understanding is already stating their difference, since the first A is subject, the second predicate. He perceives how understanding tries, at any price, to erase such unavoidable consequences of language. He goes after understanding, not aiming at denying its merits, as in what he calls finite sciences, like "mathematics", but aiming at obligating understanding *to go beyond itself*, demanding that "finite and infinity be reconciled in a philosophy which, by realizing its deep pledge of true unity, would be the true philosophy, *the* philosophy" (B. BOURGEOIS *in* HEGEL, 1970, p. 22).

There is no record of Hegel's discussion with his patrons during his eight year tutoring period. We only know that he resurfaced with the project that he developed during the rest of his life. We can speculate about him, placed between teenagers and their parents, trying to educate under mandate. We only know the result. In the mentioned book, he takes this proposition A=A as meaning simultaneously a *stated identity* and an *uttered difference* and inserts it at the basis of speculative philosophy, claiming for the *identity of identity and non-identity*[9]. He conceives of Logic as the science of pure thought; however, Bourgeois reminds us, "the forms of thought are initially exteriorized and deposited in the human language". Therefore, speculative philosophy as exposed in Hegel's Encyclopedia, whose first volume is Logic, may be taken as *the* philosophy of language ─ which Hegel calls mediation. Of course, language is here understood in its wide sense, beyond spoken words, including any representations of a human subject by a signifier to another signifier, according to Lacan; the speech of parrots is not language. Today, the agora includes electronic publications such as this one, which the reader has in hand. Such self-reference is not to be avoided; it must be included as grounding for speculative philosophy.

Recalling Marx's aphorism about the anatomy of man and the primate, we can say that the class-struggle that today takes the acute form of capital against labor, was present under a milder form in Ancient Greece: the philosophers' conservatism was challenged by the sophists. Today, as in Antiquity, the ideological struggle is rooted in economy: sophists charged for teaching the way to success. The question is whether we can identify understanding as being politically conservative. In this sense, we may argue that speculative philosophy is rooted in the sophists' movement from Ancient Greece. It seems that only at maturity did Marx recognize this class struggle position of Hegel's

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[9] Gilles Deleuze did the opposite, claiming for the *difference* of identity and non-identity, of course, after erasing his tracks by misrecognizing and negating Hegel.

speculative philosophy, and thus declared himself a disciple of that great master. Engels never did the same.

Today this understanding supports the assurance that capitalist society is the only one possible, by imposing identity over difference in the agora, which is monopolized by the official media. We feel confident that we can exercise speculative philosophy to infirm rightist positions.

6 Practicing speculative philosophy

According to the current traditional history of "mathematics",

> the fundamental theorem of calculus relates differentiation and integration, showing that these two operations are essentially inverses of one another. Before the discovery of this theorem, it was not recognized that these two operations were related.[10]

A speculative critical reading of this fragment would go as follows. The "discovery" like the discovery of America, found something that had been there from the beginning of the world, although disguised in "rudimentary form". The fundamental theorem of calculus is a trans-historical entity. From the fragment, we can also infer that only at the moment (when?) of discovery, did people realize that the operations of differentiation and integration that had putatively been well know prior to that date, were related to each other.

Does our reading not make sense, or is it the text that does not make sense? Neither, we argue. The text enunciates a proposition that once uttered denies itself. This congenital effect of language cannot be entirely avoided. It can only be minimized, in two ways. First, through what we call *quilted speech*[11] (CABRAL, BALDINO, 2020) a form of speech adequate to "mathematics", where meaning is subjected to control at each step. Strict formal logic discourse is an example. Second, in the opposite direction, in a discourse that hides its quilting points, the minimization is attempted, for instance, in Lacan's discourse, as in an impressionist-style picture.

Hegel would say that this fragment from Wikipedia is a typical discourse of *understanding*. Understanding fixes distinctions into independent entities that exist in and of themselves, and is unable to consider them as resulting from processes. For instance, an acute form of political discourse of understanding would say that wealth and poverty

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[10] https://en.wikipedia.org/wiki/Fundamental_theorem_of_calculus#History

[11] In the upholstery industry, the *quilting point* is the stitch that transforms a sack into a cushion. Lacan uses this image to refer to the moments in a discourse that retroactively assign meaning to what has been said.

exist in and of themselves and that if the poor were eliminated, the world would belong to the wealthy. Forests can be replanted, ignoring that extinct species cannot be resuscitated; the victims of earth sliding in the favelas of Rio de Janeiro are to be blamed for living there, etc. At a higher level, this form of thinking considers social classes as independently constituted; they eventually meet and clash. Understanding does not see class struggle as a process rooted in daily production. This "blindness" is perhaps its best-guarded stronghold.

The *speculative philosophy* proposed by Hegel is characterized by taking into account the statement as well as its unavoidable negation when it is uttered; following this negation, a movement occurs in the realm of language that Hegel calls *becoming* (*Werden*); it encapsulates the result of mediation back into the form of a statement, now including a negation of the first negation.

Therefore, the basic rational unit of speculative philosophy consists of three inseparable categories: statement-utterance-becoming, or thesis-antithesis-synthesis. Speculative philosophy opposes understanding, but not as an understanding-like opposition, as two independent philosophies existing in and of themselves that must eventually meet and clash. Speculative philosophy opposes understanding by transcending (*aufheben*) it, by trying to push it beyond itself, by trying to lead it to recognize the negative side of its utterances and realize their unity as sides of the same process. Of course, the success of speculative philosophy means the end of understanding, but also the end of speculative philosophy itself, since it would be left without the object of its practice.

7 You cannot because you ought

The next seven pages comprise the backbone of this article. They will probably offer greater difficulty since they are based on a line by line reading of Hegel. We intend to show how the following challenge by Bernard Bourgeois can be faced.

> Many people have passed beyond Hegel but without actually passing through him; surely, in the case of Hegel, it is easier to pass beyond him by claiming to understand him better than he understands himself, than by undertaking the threatening work of trying to understand what he has effectively said. (B. BOURGEOIS in HEGEL, 1970, p. 8).[12]

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[12] On a beaucoup dépassé Hegel, mais en vérité sans passer par lui ; assurément dans le cas de Hegel surtout, il est plus facile de le dépasser en affirmant qu'on le comprend mieux que lui, que de passer par le si redoutable travail de chercher à comprendre ce qu'il a effectivement *dit*.

Hegel's inversion of Kant's famous ethical precept *you can because you ought*, stems from a simple effect of language, for in saying *ought* "limitation is equally implied" (HEGEL, 1969, p. 133) so that "ought not" immediately becomes a possibility. It is interesting and fundamental for us to understand how Hegel derives *ought* from logic, the science of pure thought, or the science of language. Once we are able to assign a referent to his general rather than abstract discourse, we are able to follow it line by line; it becomes clear and even somewhat repetitive. To give an example of our assertion, we will show how Hegel's discourse applies *ipsis verbis* to a mathematical theorem. We will consider "*Barrier and the Ought*", a subsection of the *"Doctrine of Being"*, (HEGEL, 1966, p. 144) and apply its first two paragraphs to *Barrow's theorem* (BT)[13] (BARROW, 1976, p. 78) where calculus is supposed to have begun. Perhaps we should warn the reader that whenever we refer to Newton, Leibniz, Cauchy, Weierstrass and Lagrange, in addition to the historical human beings, we will mean their epistemological positions, also held by many other mathematicians.

In the preceding subsection, Hegel characterizes *the finite* as that which is transitory and ends in nothing. However, to this nothing, an existence is granted in speech: it is the nothing of the finite being. In this sense, "perishing, or nothing, is not the last word, but perishes too" (HEGEL, 1966, p. 144). The contradiction of "the finite" is that, insofar as it ends, its end ends too, leaving a nothing that *results from mediation*, that is, from the mediation that occurs in the agora. Usually, Hegel incorporates the result of mediation back into the being that underwent mediation. In the case of a finite being, he says that the finite being "collapses into itself (…) and stands perpetually opposed to the infinite" (HEGEL, 1969, p. 131). This being, returning from mediation to itself, Hegel calls *being-for-self*. Insofar as this being-for-self proceeds its logical development, he calls it *being-in-self*.

Finiteness is exemplified by the philosophies derived from Kant that Hegel calls *understanding*. Speculative philosophy, which he calls *reason*, struggles to push understanding beyond itself; understanding is a finite being that stands perpetually opposed to reason which is its infinite other. According to (HEGEL, 1801, sec. 2, p. 10),

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[13] BT, with proof and a figure will be the object of section 8. In a concise way we can forestall its statement: if we draw a line through a point of the graphic of a curve whose ordinates are equal to the accumulated area under the graph of a second curve in such a way that the inclination of this line is equal to the ordinate of the second curve a the point of equal abscissa, then that straight line will be tangent to the first curve.

along with the end of understanding, speculative philosophy also requires the end of Reason.

As another case of finiteness, we will take BT. In a previous version of this paper we had copied the mentioned subsection where Hegel considers "Barrier and Ought" and replaced "something" with "BT", but the text became too difficult to follow. In the present version we have omitted the quotations to that section by Hegel, retained only our comments.

In his 1670 proof of this theorem, Barrow (1630-1677) only uses geometric and analytic arguments, like the Ancient Greeks did. However, in the wake of capitalist economy, such finitary methods were meeting their end as a means to determine tangents to curves and areas of regions bound by curves. Therefore, we feel entitled to consider BT as a finite something.

Hegel begins the section *Barrier and Ought* referring to the previous paragraph from which we derive that as BT is finite and perishes. It remains to be seen what moments are contained in this notion of the finite BT. This will be our work in the following pages. Hegel reminds us of the previous development of finite something, emerging from mediation and encapsulated into being-in-self, from which the finite something continues its logical development. Therefore, as argued above, we assume that BT has already undergone the logical development that places it as something finite.

The determination of BT is its use of the finitary method of the Ancients that admits only analytical and geometrical arguments. By "modification" (*Beschaffenheit*) we should understand "that element within Something that becomes an Other" (HEGEl, 1966, p. 136). BT's modification lies in three shortcomings that will change along its logical development. First, Barrow's concept of tangent was incipient for the needs that came after him; he considered it as a straight line that has one point in common with a curve, the curve lying entirely on one side of the line near the point. Second, Barrow's proof is restricted to strictly increasing or decreasing functions. Third, Barrow found a very peculiar way to disguise the representation of areas by segments of straight lines, which constituted a heresy to the finitary classical method admitted as valid at his time: he postulated a constant segment to be imagined as forming the side of a rectangle whose other side would be a segment proportional to the area to be represented.

Actually, the representation of areas by lengths of segments only requires the establishment of a scale factor, but this would have made the proof depend on number, rather than on geometry. This is a typical example of *epistemological obstacle*, according

to Bachelard (1980). This obstacle points to the primacy of method with respect to result: only the finitary method of the Ancients was admissible to Barrow and they continued so for Newton, Leibniz and the others, originating the crisis of calculus. Apparently, numbers were not considered noble enough, despite the work of Kepler (1571-1630), who had died 40 years before.

The BT logical limit consists in a stoppage. Barrow stopped at the point where he could have shrunk the so-called characteristic triangle[14] towards the point of tangency to obtain the *inclination of the curve* in an infinitesimal neighborhood of the tangency point or as the mathematical limit of the inclination of the tangent line. Refusing to go beyond the finitary method, he provided us with an example where the *mathematical concept of limit* was also the *logical limit*. Beyond this common limit lies the terrain where Newton and Leibniz were at home, Leibniz receiving the extra merit of having invented the notation used today. In fact, they made small additions to BT, with timid reference to Barrow.

BT was negated by the action of Newton and Leibniz. This first negation consisted in expedients for calculation dealing directly with the infinite; they traversed the logical limit that had refrained Barrow. For the next century mathematicians were unable to justify their method for dealing with the infinite, which relied on the concept of mathematical limit and on infinitesimals; during that time, the polemics inaugurated by the criticism of Berkley in 1734 grew in quantity, but remained stuck in quality.

By accusing the mathematicians of his time of being unable to provide a justification of their method, Hegel's criticism should count as a negation of the mathematicians' method inaugurated by Newton and Leibniz and that still persisted the 18th century. This method, which we call *infinitesimal calculus* (IC), was already a negation of BT; therefore, the criticism of Hegel would have been a second negation, a negation of the negation of BT-IC, producing a determined result in the agora, a being-in-self. Since Hegel considers infinitesimals a chimera and the mathematical limit as the bad infinite of understanding, we expected him to complete this second negation of BT by negating BT-IC. But he never did, as we will see shortly.

In Hegel's time, the current history of mathematics community was at a crossroads; most of the troop chose the path that followed Lagrange (1736-1813), Cauchy, who became the hallmark of this path, and later on, Weierstrass, with his

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[14] The right-angle triangle having the hypotenuse on the tangent line and sides parallel to the coordinate axes.

philosophic disciple, Edmund Husserl. Turning his back to the concept, Cauchy (1824) found a way to deal with the infinite, by cutting off the polemics at the level of language. For instance, in $\lim_{n\to\infty}(1/n)=0$, through a semantic convention he skipped over the problem of what happens when *n* grows indefinitely: *one says that the limit is zero and that the sequence becomes infinitesimal*. [15] Thereby Cauchy got rid of Newton's drama contained in the concept of mathematical limit: what happens at the moment 1/*n* reaches zero, etc. This language convention by Cauchy opened the door for Weierstrass, to later completely finitize the bad infinite with his epsilon-delta theory. Mathematicians abandoned any concerns with the foundations of their method in favor of conventionally valid results. From the pint of view of speculative philosophy, in M20 the thesis is already contained in the hypothesis and the prof is tautological.

Apparently, Hegel stood alone at the outset of the other path; he pleaded with people to justify the mathematicians' method via speculative philosophy, since the discussion was still hot at his time. It is not evident from a first reading of Hegel, but if we look closely, we will find him beside Lagrange, being followed by the troop, as we will see. The political consequences of the Cauchy-Weierstrass *tour de force* will be dealt with in the final sections of this paper. In this section, we will contribute to complete the second negation of BT-IC that Hegel did not do.

Now we continue our reading of the two intended paragraphs of *Barrier and Ought*. The finitary method as a determination of BT and the three shortcomings comprising BT's change, or "quality", were additions made by our external reflection, from outside our object of study. For the sake of theoretical rigor, it is necessary to show that what we introduced matches what was already in the nature of something, namely, BT. We will address this next.

The externality that we introduced into BT coincides with the points where Newton and Leibniz anchored their first negation of BT. By using Barrow's efforts in order to justify their own method through finitary arguments, they produced a modification as they surpassed the incipient definition of the tangent line; they also surpassed the restriction to monotone functions in the proof and were not ashamed of comparing areas using line segments. This satisfies the theoretical requirement for rigor, since we find the additions of our external reflection already present in BT; however, this

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[15] Actually, Cauchy reasoned with infinitesimals. In Sad, Teixeira and Baldino (2001) we showed that what Cauchy wrote is perfectly correct if we read it from the point of view of non-standard analysis, developed by Robinson in the 1960s.

externality is still ours, that is, we may still use it for eventual new additions to our object of study.

The logical limit and the mathematical limit determined both, BT and its other. By crossing this limit, IC provided a first negation of BT, a negation of the finitary method where Barrow had stopped. A second negation means mediation in the agora. This was started by the new infinitary methods of IC. The otherness is inherent to BT but still remains in us, and relates the two sides, namely, the finitary method and the three shortcomings of BT. However, now in the agora, the finitary method of the Ancients remained as the hallmark of IC's attempt to justify the bad infinite: how could the infinite be explained in terms of the Ancients? This means that our object of study has been modified; it is now *the arguments that seek to ground the surpassing of the three outcomes on finitary arguments.* We have before us, to be mediated in the agora, BC-IC and its failure of self-justification. As a consequence, we have threaded ourselves into the 18th century polemics on calculus. From this point on, we are with Hegel, looking two-hundred years into the past and trying to complete the negation of BT-IC. This will be *speculative mathematics*.

Let us see where Hegel stopped. We have seen that Bishop Berkeley's attack of 1734 is the hallmark of the polemics. His attacks were based on the same metaphysical concept of mathematical infinite underlying the new method, namely, the mathematical limit. Hegel depreciates this concept as the spurious infinite, and appreciates the concept of infinite that mathematicians used, but reproached them for their inability to use it in their own defense.

In our first reading, we imagined that Hegel's criticism of IC would lead to its negation in the agora. However, a closer reading of Hegel shows that we must distinguish what he said should be done from what he effectively did. Criticizing the way that the philosophies of his time considered their object, Hegel says: "Instead of dwelling within it and becoming absorbed by it, knowledge of that sort is always grasping at something else; such knowledge, instead of keeping to the subject-matter and giving itself up to it, never gets away from itself" (HEGEL, 2012, p. 4).

We would expect Hegel to abandon himself to the polemics around IC. Instead, he apparently tried to respond to his own criticism: despite the success of the use of the mathematical infinite, "this science has not yet succeeded in vindicating this use of it conceptually" (HEGEL, 1966, p. 256). After describing several mathematicians' attempts to justify their method, he concludes:

> It has been shown in the preceding Observation how idle an attempt it has been
> to find principles for the former manner of comprehending the procedure —
> principles which really solved the contradiction which there was found (…)
> the insignificance of that which was to be omitted (…) or infinite arbitrary
> approximation and the like (HEGEL, 1966, p. 294).[16]

From this point on, Hegel prepares the terrain to appreciate Lagrange's method of power series: "It was Lagrange who rejected this pretense [the characteristic triangle] and followed the true scientific course" (HEGEL, 1966, p. 305). At his time not everyone knew that there are smooth functions that cannot be developed into power series, such as e^{-1/x^2}, which restricts Lagrange's method to the so-called analytic functions. The necessity of examining the convergence of the power series reestablishes the bad infinite into what Hegel thought would be the last word on IC.

Recognizing that the mathematical infinite cannot be justified "by the notion", Hegel did not negate IC, but showed its finitude, insofar as it relies on the bad infinite of mathematical limit. Indeed, IC ended soon after, with Cauchy and Weierstrass. It is presently called *differential calculus*. Today, "IC" designates the calculus of infinitesimals proposed by Abraham Robinson in the 1960s, but this is a post-Weierstrass calculus, entirely dependent on the bad infinite. Hegel did not dwell within his object as he recommended, and he did not arrive at elevating the spurious to the true infinite. Therefore, his criticism does not amount to a negation of BT-IC. Together with Newton and Leibniz, his criticism remained at the level of the first negation of BT, with IC counting as a modification of BT. The negation of BT-IC as a finite Something is still to be done. This will be our endeavor.

BT-IC has both determination (a drive for finite methods) and modification (the negated shortcomings of BT plus introduction of the bad infinity). The BT-IC quality is the success of its applications, along with the lack of conceptual foundation. The limit of BT-IC that determines its other and has to be mediated in the agora is now the bad infinite on which the mathematicians' method relies, before and after Hegel, up to the present time. In the agora, BT-IC self-identity became an introverted relation to itself; the negation of the limit *now comes from inside BT-IC itself*. Therefore, our negation of BT-IC consists in nothing more than dwelling within it and *letting it speak for itself*. Here we invite Hegel to join us, so that we can finish together what he left as homework. We introduce our friend, Mr. John Q. Understanding (JQU) to Hegel.

JQU: What do you mean by letting BT-IC speak for itself?

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[16] Apparently, he as not satisfied with his own solution, consisting of considering the vanishing magnitudes as intensive ones.

> H: You have just uttered its first sentence.
> JQU: I don't understand.
> H: You referred to "it", didn't you? You are making "it" alive; BT-IC is coming into being through you. You are announcing "it".
> JQU: I could have chosen to say nothing.
> H :   If you choose to say nothing about "it", that will be "it"'s silence.
> JQU: I can say absurdities about this damned "it" (irritated).
> H: If you know what an absurdity about "it" is, you also know what is pertinent to "it". Your uttered absurdity will also reveal "it"'s pertinence as negated.
> JQU: With smart absurdities I can make it be whatever I want.
> H: Of course, as long as you are the only reader of this paper… the truth of BT-IC, like the truth of anything, results from the agora. It is never all. *The path to truth is already the truth*; it is paved with signifiers *and their negations*. In the agora, it will be hard to distinguish whether people are talking about BT-IC or whether BT-IC is talking through them. Being and thinking are the same.

The addition that IC introduced to BT through its mediation in the agora implied that, thenceforth, it was no longer sufficient to find solutions to problems, for instance, like Cavallieri and Torricelli had done with their indivisibles[17]. Solutions now had to be justified. The passage from one step to the next became the object of scrutiny; it had to be put into words, justified. The form of discourse used by the Ancients to justify their finitary method had to be extended to include the treatment of the infinite. In Cabral & Baldino (2020) we called *quilted-speech* (see footnote 10) a form of speech where the sliding of the signified under the signifier is kept under control. This form of speech emerges from a dialectics in history, together with a community of speech that decides what counts as a valid argument at each cultural epoch. Hegel fell into the trap of looking for a quilted speech that would justify BT-IC by the concept, and believed that Lagrange had uttered it. Actually, quilted speech is the pinnacle of understanding. Cauchy and Weierstrass only reached it by using the semantic convention "*on dit que*". Back at the crossroads, the troop followed them, and eventually arrived at M20 and its community of identity-quilted speech, the *chef d'oeuvre* of understanding.

Words like "Being-in-Self", and "negation of negation", appearing in the final part of *Barrier and Ought*, at the end of Hegel's two paragraphs we're commenting, indicate the logical operation resulting from mediation. The result of mediation is taken back into the self-identity of BT-IC through the logical operation that Hegel calls "wrapping up" (*eingehüllt*) (HEGEL, 1966, p. 145). As an encapsulated Being-in-Self, BT-IC still retains its shortcomings and its infatuation with the finitary method. However, following its logical development, now it also contains the mediation as quality of its

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[17] They found, for instance, the area of a parabolic segment of any degree, which the Ancients only knew for second degree.

being-for-self. Reaching that final stage, Hegel will be able to leave the mediation to the entities that he would have elicited, let speculative philosophy speak for itself, and leave the scene. Following this path, Marx left us Marxism, before he died in 1883, and after having recognized himself a disciple of Hegel.

We do not have to carry out this final compacting operation, since we assume ourselves in the agora, in an action of mediation that continues under the form of our intervention into the political-philosophical scene. We do not have to turn our object of study into a subject and leave the scene. As Marx did in life, we remain in action in the agora.

Now it is BT-IC itself that, returning from mediation in the agora, posits its limit as negated and determines it as an essential barrier. Hegel will develop essence in the second book of Logic, where he will say, in short, that essence is spoken being, or the discourse of Being. In the agora, BT-IC is now universal; not yet a subject, but in the process of becoming one. Hereby its limit follows suit and universalizes itself, passing from limit to barrier: the bad infinite becomes universalized and, as such, becomes object of language, of mediation.

What is posited as negated is not only barrier, beyond which lies the true infinite; what is posited as negated is the limit itself, the bad infinite of understanding, and this is what is divides BT-IC and us. Since at this stage of Hegel's text encapsulation has been assumed, this negation stems from the in-itself of the determination of BT-IC, namely, from the finiteness of its method, from its finitude. In the universalization resulting from encapsulation, the limit ─ the bad infinite ─ becomes the Being-in-Self of determination ─ the finitary method. In summary, the negation contained in the wrapped up Being-in-Self negates both the bad infinite and the finitary method used thus far to try to justify it. The old problematic of justifying the methods of calculus is surpassed (*aufgehoben*) and the true infinite lies in front of us.

In the end of the two paragraphs under consideration, Hegel concludes: "This Being-in-Self is then the negative relation to its limit (which is also distinct from it), or to itself taken as Barrier: that is Ought" (HEGEL, 1966, p. 144). That is, to carry out the negation of the bad infinite and the finitary method, transcending both, is an *Ought*. Surprisingly, a precept for action is dug out from pure logic. It stems from the tension resulting from the encapsulation process: the being-in-itself contains the negative relation to its limit, i.e. BT-IC contains the *negation of its own finitude*, not abstractly or externally, but finally resulting from mediation in the agora.

In our case, being-in-self of BT-IC is now universalized and contains speculative philosophy in the form of quilted-speech, a discourse suited for mediating mathematical results. However, we will not refrain from completing the final encapsulating step, and we will not leave the scene; we remain active, in the agora. Our presence is the other of BT-IC; we assume the tension in ourselves, the tension that we refused to encapsulate in the being-in-itself. So our practice *ought to negate* the finitizing practice of understanding carried out by Newton, and Leibniz – and by Hegel himself, insofar as he did not follow his own philosophy. This practice extends itself to the present form of BT-IC which is M20 and the M20 community.

Our practice includes itself in the general process of the struggle of reason to push understanding beyond its limit and to seek to *incorporate the so-called invalid methods of calculus into validity itself*, thereby beginning a *new history* of the M20 community that we call *speculative mathematics*. Hegel's Ought, derived from Logic, turns out to be the old class struggle that determines understanding and reason as opposite poles of a dialectics that has always been called philosophy.

8 The theorem of Barrow

There are two common ways of presenting mathematicians' antique writing. One way is to scan an edition, as antique as possible, and display it, plainly stating the modern result equivalent to it. Another way (Baron, 1985, p. 45) is to transcribe the antique text and introduce explicative notes at key points. We will use a third method; we will write the statement and the proof in modern language and elicit the points to which this article refers. We will preserve Barrow's form and notation, with some modern additions.

Let **R** be the length of a fixed line segment. Let ZGE be the graph of the function $z = f(x)$ with the z-axis pointing downwards as in figure 1. Suppose that the ordinates of $f(x)$ are increasing. Let VIF be the graph of the function $y = F(x)$ defined in such a way that the area of the rectangle with sides **R** and $F(x)$ is equal to the area of the region VDEZ, i.e. the area accumulated under the graph of $f(x)$: $area(\text{VDEZ}) = F(x)\mathbf{R}$.

Through point *F,* draw the straight line *TF* where $DT = \dfrac{DF\,\mathbf{R}}{DE} = \dfrac{F(x)\mathbf{R}}{f(x)}$. Then this line is tangent to the graph of $F(x)$ at the point F. (Don't be concerned with the double meaning of the letter F, for point F and for the function F.)

Proof: Let I be any point in the graph of $y = F(x)$, either above or below point F. Draw the lines IG parallel to the z and y-axes; draw IL parallel to the x-axis. Let K be the intersection of IL with TF.

Suppose, first, that I is above F. Then, in the style of quilted speech (Cabral & Baldino 2020), we have:

$$DT \overset{(1)}{=} \frac{F(x)\mathbf{R}}{f(x)} \overset{(2)}{=} \frac{DF\,\mathbf{R}}{DE} \implies DF\,\mathbf{R} \overset{(3)}{=} DT\,DE \implies \frac{DF\,\mathbf{R}}{DT} \overset{(3)}{=} DE$$

$$\frac{LF}{LK} \overset{(4)}{=} \frac{DF}{DT} \implies \frac{LF\,\mathbf{R}}{LK} \overset{(3)}{=} \frac{DF\,\mathbf{R}}{DT} \overset{(10)}{=} DE \implies$$

$$LK\,DE \overset{(3)}{=} LF\,\mathbf{R} \overset{(5)}{=} (PI - DF)\mathbf{R} \overset{(3)}{=} PI\,\mathbf{R} - DF\,\mathbf{R} \overset{(6)}{=}$$

$$= area(VPGZ) - area(VDEZ) \overset{(7)}{=} area(DPGE) \overset{(8)}{>} DP\,DE \implies$$

$$\implies LK\,DE \overset{(9)}{>} DP\,DE \implies LK \overset{(3)}{>} DP \implies LK \overset{(11)}{>} LI$$

(1) Definition. (2) Notation. (3) Algebraic transformation. (4) Similarity of triangles and *FLK*. (5) Addition of lenghts. (6) Definition of $y = F(x)$. (7) Superposition of areas. (8) Assumption that $y = f(x)$ is increasing and *I* is above *F*. (9) Collect previous chain. (10) Previous result. (11) Equal lenghts.

If the point *I* is below *F,* this argument will lead to $LK < LI$. Together with the previous result, this shows that curve $y = F(x)$ lies above the line *TF*. Barrow concludes that *TF* is the tangent line.

For the purposes of this article, we should stress:

1) Barrow did not say that $\frac{FL}{LK} - \frac{FL}{LI}$ tends to zero or becomes infinitesimal when respectively FL tends to zero or becomes infinitesimal. Stopping at this point, he refused to extend the concept of ratio from straight lines into curves. This is Barrow's stoppage to which we referred in the previous section.

2) The introduction of **R** allows the argument to remain at the geometrical level: areas are compared to areas. Since the figure is just a diagram, an adequate assumption of the scale of the ordinates of $F(x)$ would make **R**=1. This restriction would have made the proof to depend on number, a realm that had scared mathematicians since Pythagoras and that apparently constituted an epistemological obstacle in the time of Barrow. This is Barrow's trick, to which we referred in the previous section.

3) Being all on one side of the line, TF does not ensure the usual concept of tangency, as shown by the function $y = |x|$ and the x-axis; the graph of $F(x)$ may have an angle at F. To avoid this, the differentiability of $F(x)$ should be proved, but this depends on the continuity of $f(x)$ and the proof requires either limits or infinitesimals. This is the incipient concept of tangency to which we referred in the previous section.

4) If $F(x)$ had a local maximum at E, the result would still be valid, but the proof would have to be modified. This is the restriction to functions that are either increasing or decreasing, to which we referred in the previous section.

Figure 1: the theorem of Barrow

9 The infinity of quantum in weighing a slice of mortadella

John Q. Understanding (JQU) and Hegel have brought a sliced mortadella into the laboratory. JQU intended to obtain its weight by using only the small laboratory scale, limited to 100 g. JQU has just weighted the first slice.

> JQU: This small one weighs 14.4 g.
> H: *Moment mal.* Are you sure? Could it be $10\sqrt{2}$?
> JQU: Well, on a more precise scale we could get more digits, if that is what you mean.
> H: Yes. How much more precision could you get?
> JQU: Are you kidding? You can go down into the atomic level, for quantum determination.
> H: Well! You are starting to realize that determination of this quantum of mortadella is not so simple. Please describe what you should do to determine this quantum, for the sake of my scientific record.

> JQU: Well, I also noticed that that the scale pointer oscillated a little, before it stopped at 14.4.
> H: Precisely. When a merchant wants to weigh one kg of flour, say, he has to add and remove a little bit from the package to adjust the scale pointer to 1 kg. Weighing oscillates…
> JQU: So he never gets quite the right weight, is that what you trying to get at?
> H: Not quite. What I am saying is that what you call "the right weight" is an infinite process. The infinite process is just another quantum.
> JQU: (Showing some irritation) Fortunately, we do not have to go through your fancies. Register in your notepad that I am taking 14.7 g and let's go on.
> H: Ok, "we now have quantum determined in conformity with its Notion, which is different from quantum in its immediacy" (HEGEL, 1969, p. 239).

JQU only sees the quantum as it appears to him, and neglects the process that led him to transcend (*aufheben*) what he called Hegel's fancy and assume 14.4 g as simply given. Rounding off is a logical operation. For Hegel, determination of the quantum $\sqrt{2}$ is outside $\sqrt{2}$, in the bad infinite that today we describe as the sequences that converge to $\sqrt{2}$.[18] These sequences constitute another quantum that Hegel calls the externality of $\sqrt{2}$. However, the externality of $\sqrt{2}$ was already the negation of that immediate form that JQU attributed to 14.4… Therefore, the ultimate determination $\sqrt{2}$ is *negation of its negation*. In the dialogue with JQU, $\sqrt{2}$ and its externality were transcended by mediation in the agora, forming one *qualitative* unity. The determination of quantum lies in another quantum "through the mediation of its not-being, namely, of infinity; that is, it is *qualitatively* that which it is" (HEGEL, 1966, p. 255). In summary, quantum is *qualitative infinity* and this has nothing to do with an infinitely great or infinitely small magnitude. The quality of $\sqrt{2}$ as the diagonal of a square with sides of length one differs from the quality of π as the length of a circumference of radius one.

The moment that the reader is now living in the agora is the continuation of the above dialog into the determination of the quality of $\sqrt{2}$. This determination will never be complete. The identification of $\sqrt{2}$ with a cut in the rational numbers, or with a set of so-called Cauchy sequences, is just another moment of the same determination.

10 The bad and the true infinite

> The mathematical infinite is interesting, first because its introduction has widened the scope of mathematics and has led to important results therein; next it is remarkable because this science has not yet succeeded in vindicating this use of it conceptually (HEGEL, 1966, p. 256).

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[18] We should keep in mind that the identification of √2 with a set of sequences only occurred one hundred years after Hegel.

This fragment contains a project of political epistemology that has never been developed. It requires that the difference between the bad and the true infinite be well understood. We invite Hegel and our friend John Q. Understanding (JQU) to tell us the difference.

> JQU: From my German class, I have learned that finite is *Endlich* and infinite is *Unendlich*. These names are saying *endness* and *un-endness*, if these were English words. Therefore, the finite is that which ends, the infinite is that which does not end.
>
> H: I see. Finite and infinite oppose themselves through the notion of ending. Can you give me an example?
>
> JQU: Yes. I may count up to ten and stop. This is finite. Or I may go on counting. This is infinite.
>
> H: Tell me, when you count, are you doing the same operation from one number to the next or are they different operations?
>
> JQU: It is the same operation; I am just adding one.
>
> H: It seems to me that there is a difference. When you count up to ten, say, you distinguish the last operation. This distinction determines your counting as finite and, simultaneously, brings to the mind the idea that it could go on, hence the infinite. Right?
>
> JQU: I suppose.
>
> H: But if you go on forever, each counting operations is not distinct from the preceding one anymore. Being unable to logically distinguish one operation from the other, you are actually doing nothing, or you did it only once. Hence, you stopped. Your finite has passed through the infinite and returned to the finite.
>
> JQU: There you come with your sophisms. You will certainly not let me use time as a distinction; so, I must agree. I have entered an endless loop.
>
> H: Precisely. What do you do when a software enters an endless loop?
>
> JQU: I turn it off. The machine is not supposed to enter that loop.
>
> H: Yes, it is supposed to do something else, right? What is it supposed to do?
>
> JQU: I don't know; it depends, anything, I suppose.
>
> H: Voilà! You cannot say that it entered a loop without bringing to mind the idea that something else was expected. In principle, this something else is the other of the endless loop, it is one among infinite possibilities of stopping. Your infinite has passed through the finite and returned to the infinite.

It does not matter from which one, the finite or the infinite, we start, mediation in the agora takes us back to the start. This unavoidable effect of language determines the infinite as a unity of finite and infinite which is called the true infinite. The counting on is the bad infinite.

> And as both finite and infinite are themselves moments of the progress, they jointly are the finite: since jointly they are negated in the progress and in the result, their result, which is the negation of the finitude of both, is justly called the [true] infinite. (HEGEL, 1966, p. 161).

11 The M20 community

Hegel distinguishes three forms of infinite. The bad or spurious infinite is what today the mathematics community calls sequences; these are functions whose domain are the natural numbers. The mathematical infinite is determined as "a magnitude that cannot

be increased". The true infinite is the transcendence of both, the finite together with the bad infinite, through mediation in the agora and encapsulation back into a unity, as a Being-for-Self.

Magnitude (or *quantity*) is a determinateness that has become indifferent to the being to which it refers. "If a field (for instance) changes its [quantitative] limit, it remains what it was ─ field. Whereas if its qualitative limit is changed (…) it becomes meadow, forest and so on" (HEGEL, 1966, p. 199). *Quantum* is a determinate magnitude; it is determinate by other quanta that make its otherness and constitute its quality. We have already shown that the determination of the weight of a slice of mortadella is an infinite process. According to its concept, quantum is intrinsically infinite. In its complete determinateness, quantum is number[19]. Therefore, *a natural number is an infinite process* encapsulated into a being-for self.

The mathematical infinite, at Hegel's time, "was determined as a magnitude greater than which there is no other" (HEGEL, 1966, p. 258) and similarly for infinitesimal magnitudes.[20] However, "a magnitude is defined in mathematics as something which can be increased or decreased" (HEGEL, 1966, p. 258). Consequently, an infinite magnitude is no magnitude at all, much less a quantum. In this sense, of finite/infinite magnitudes, "infinite quantum" is a meaningless expression, "finite quantum" is a redundancy. For nearly two hundred years this imbroglio caused people to lose their sleep.

> This is just what constitutes the difficulty for ordinary thought; for it is demanded that Quantum, in so far as it is infinite, be thought of as transcended and as something which is not a Quantum, although its quantitative determinateness remains. (HEGEL, 1966, p. 259).

Finally, Cauchy said *enough!* He refused the debate and invited his followers to quit the agora, or to form a kind of autonomous closed sub-commission that came to stabilize itself in the beginning of the last century as the M20 community.

> When the successive numeric values of a variable steadily grow in such a way as to surpass any given number, one says that this variable has the positive infinite as limit, denoted by the *sign* ∞ if it is a positive variable; and the negative infinite denoted by the *notation* -∞ if it is a negative variable (CAUCHY, 1823, p. 4, our emphasis). [21]

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[19] Here we should understand natural and rational numbers. At the time of Hegel the straight line was not identified with what came to be called real numbers. We contend that Hegel was a precursor of Dedekind, but this question is still under debate.

[20] Comparison of infinite magnitudes only started much later, with Cantor; however, they were defined in terms of the Cauchy-Weierstrass way of dealing with the bad infinite and do not compare with the infinite magnitudes of Newton and Leibniz's time.

[21] "Lorsque les valeurs numériques successives d'une même variable croissent de plus en plus de manière à s'élever au-dessus de tout nombre donné, on dit que cette variable a pour limite l'infini positif indiqué par

Curiously, Cauchy thought in infinitesimals. In Sad, Teixeira and Baldino (2001), we showed that what he wrote makes perfect sense if read in the light of non-standard analysis. By taking refuge in their bunker, Cauchy, Weierstrass and their followers locked out the true infinite, including the natural numbers which, as we have seen, are infinite processes. They only admitted the bad infinite, finitized by the Weierstrass epsilon-delta theory. The bad infinite became the only principle that the M20 community admits as a valid criterion of quilting speeches that naturally refer to the true infinite. *M20 does not know what a number is.* M20 granted nobility to the bad infinite by calling it *recursion* and made it the cornerstone of M20 certainty.

12 Speculative mathematics

Reason does not struggle *against* understanding; it struggles to *transcend* (*aufheben*) understanding. Speculative mathematics does not struggle against M20, the *chef d'oevre* of understanding; it struggles to *transcend* the M20 community, to bring it back to the agora and push it to its beyond. This struggle is dichotomic but not symmetrical. M20, as a community of identity-quilted speech, is a finite branch of understanding. It is bound to end. Pure, or rather unapplied, mathematics, based solely on the spurious infinite, may soon turn out to be an unproductive activity, that is, unable to produce surplus-value and stir up the market. In this case, speculative mathematics will end too. However, while the M20 community conserves its power, it will keep dictating rules to other practices, especially to mathematics education. The so-called *new math,* in the wake of the 1958 *sputnik* is a good example, albeit not the most harmful one.

Speculative mathematics is not a new M20. It is not a new way of doing "mathematics", it is not a new ground for M20 certainty. It only pushes the M20 community to recognize its shortcomings: *it does not know what a number is*. It does not know what it means by "mathematics". It does not recognize the true infinite, and identifies the *presence* of the transcended quantum with the *place* pointed at by the process. By brute convention of language, the M20 community discards the dialectics of place-and-presence that could justify its bad infinite process.

> Castration introduces the distinction between an element and its (empty) place, more precisely: the primacy of the place over the element; it ensures that every positive [non incestuous] element occupies a place which is not

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le signe ∞, s'il s'agit d'une variable positive ; et l'infini négatif indiqué par la notation - ∞, s'il s'agit d'une variable négative". Clearly, negative numbers were not yet in course.

"consubstantial" to it, that it fills out a void which is tot "its own" (ŽIŽEK 2002, p. 231).

On dit que each real number is a half line of rational numbers stuck into a "cut". The gaps in rational numbers are filled in with half lines of the same rational numbers. This "raising of itself by the bootstraps" is a closing of the gap by brute force, parallel to the closing the social gap by the brute force of totalitarianism. "We should recall that the [Brazilian] mathematicians did not participate of opposition movements to the military governments that, in the scientific area, were concentrated in the SBPC".[22]

Speculative mathematics challenges M20 as a *community of speech* based on language conventions and the political position to which this practice leads. To further determine the concept of a community of speech, we will repeat *à vol d'oiseau* the history of M20 as we narrated it in Cabral and Baldino (2020).

Due to special circumstances, in Archaic Greece (800-480 BC) the social conflict of rich vs. poor became an intra-family clash between the progenitor and his younger brothers, enriched in commerce. This conflict could not be solved through weapons; a special form of dialog was necessary: *strike but listen* said Themistocles to the Spartan admiral who threatened him physically during a Persian invasion. We call *quilted-speech* the discourse that was developed in that historical circumstance. Not only the utterances, but also the *arguments* justifying them became object of thought and control. The development of this embryo led to numeric-quilted speech, with Pythagoras, and to geometric-quilted speech with Plato and later with Euclid. It also gave rise to the team of un-quilters, called sophists, to whom we owe the first speculative philosophical practice, pushing understanding beyond itself. From the debates in the agora emerged both quilting criteria and communities of speech that decided which arguments could count as quilting points. Such dialectics generated philosophy, democracy, and a community of speech called "mathematics". From François Viète (1540-1603) on, algebraic quilting started being considered. Geometry and algebra were the quilting criteria in the 17th century. Pythagoras' numeric quilting stumbled on √2 and was suspended, to be retaken by Hilbert in the turn to the 20th century.

À vol d'oiseu we land on the crossroads where Hegel was making his choice. The troop followed Lagrange, Cauchy and Weierstrass. That road led them to M20. After a

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[22] É preciso lembrar que os matemáticos não participaram de movimentos de oposição aos governos militares que na área científica estavam concentrados na SBPC [Brazilian Society for the Progress of Science]. César Camacho, internal communication in IMPA, July, 16, 2001.

new quilting criterion, introduced by Frege (1848-1925), M20 emerged as a *community of identity-quilted speech* (CABRAL AND BALDINO, 2020). The decisions in this community are not taken in congresses through voting, as one might expect; they are surreptitious. For instance: given a collection of non-empty sets, can we form a new set, choosing one element from each of them? This is the axiom of choice that leads to conceptual paradoxes. "Despite these seemingly paradoxical facts, most mathematicians accept the axiom of choice as a valid principle for proving new results".[23] This acceptance was never object of a public decision by the community. M20 evades the agora.

We invite Hegel to come back and take the other road with us. Under the concept of speculative mathematics, the history of calculus becomes a new history, namely the history of surreptitious quilting criteria, as well as of the needs, successes and consequences of such quilting. Through this *other history* we enter today's mathematics classroom from a new door.

13 Conclusions

How many people besides Bernard Bourgeois have gone through Hegel's Logic as demanded by him? We have shown that such an effort is worthwhile; not only does a new history of the M20 community emerge but, in the horizon, lurks a Marxist Hegel.

We hope to have elucidated via concept two commonplace visions of the so-called "mathematics". It is reputed as the principal college gatekeeper for engineering and sciences courses, and it is reputed to be the most hated school discipline. We showed that these visions stem, in first a degree, from the epistemological nature of the knowledge dealt with in M20. This knowledge is based on a brute-force language convention that rules out free mediation in the agora and simultaneously imposes a finitary treatment of the bad infinite through the Weierstrass theory.

In a second degree, exclusion by, and hate for "mathematics" depends on the effect of the epistemological nature of that knowledge on the *unconscious* of people who are accountable for its production and guarantee. The unconscious, says Lacan, is the *discourse of the Other*, in this case, of the M20 community. The exclusion of the good infinite (the mediation in the agora) extends, naturally, to the exclusion of people who

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[23] https://en.wikipedia.org/wiki/Axiom_of_choice

seem contaminated by this sort of *jouissance*.[24] Hence the affinity of the M20 community to conservative positions.

A classroom action based on speculative philosophy should certainly praise the finitary geometric method of the Ancients, cast into the identity-quilted speech associated with geometry. However, beyond this, the true infinite, together with the un-quilting strategies of the so-called sophists, should also be admitted, rather than denied. Speculative philosophy should be exercised as a criticism of the conservative political position of the M20 community. The classroom should open itself to the agora and try to unlock the bunker of understanding of M20 and its illusion of superiority. In calculus courses, infinitesimal methods are certainly one of the main avenues for introducing *speculative mathematics* as a practice of *political epistemology*.

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[24] We use the French term because the English *enjoyment* fails to capture the sexual connotation.

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